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**Abstract of the doctoral thesis**

**Title:** $(1,2)$-*dominating sets in graphs*
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The concept of a $(1,k)$-dominating set was introduced in 2008 by S. M. Hedetniemi, S. T. Hedetniemi, J. Knisely and D. F. Rall. Let $k \in \mathbb{N}$. The subset $D \subseteq V(G)$ is a $(1,k)$-dominating set if for every vertex $x \in V(G) \setminus D$ there are two distinct vertices $u, v$ in $D$ such that $x$ is adjacent to $u$ and the distance between $x$ and $v$ does not exceed $k$.

The thesis deals with properties of $(1,2)$-dominating sets and parameters related to $(1,2)$-domination. In particular, exact values of the $(1,2)$-domination number were determined in certain classes of graphs. Moreover, relations between the $(1,2)$-domination number and the domination number were given.

Since every $(1,1)$-dominating set is also a $(1,2)$-dominating set, we discussed proper $(1,2)$-dominating sets, namely $(1,2)$-dominating sets which are not $(1,1)$-dominating. We solved the problem of the existence of proper $(1,2)$-dominating sets in graphs. Furthermore, we showed that in a connected, non-complete graph the $(1,2)$-domination number and the proper $(1,2)$-domination number are equal. We also gave a chain of inequalities between parameters of proper $(1,2)$-domination, $(1,2)$-domination and classical domination, which was later analysed.

Next, we studied $(1,\overline{2})$-intersection index, which is defined as the smallest possible number of common vertices of a $(1,1)$-dominating set and a proper $(1,2)$-dominating set in a graph. We determined the exact value of this index in some classes of graphs such as paths, cycles, complete bipartite graphs and spiders. Additionally, a partial characterization of trees with $(1,\overline{2})$-intersection index equal to zero was proved.

In the thesis there are also results concerning the problem of the existence of independent $(1,2)$-dominating sets in graph and their products, in particular the tensor product, the strong product, the $G$-join of graphs and the generalized corona of graphs. In each case we gave a complete characterization of the product, which has an independent $(1,2)$-dominating set.