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Rzeszów University of Technology

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### **Abstract of the doctoral thesis**

APPLICATIONS OF THE TECHNIQUE OF MEASURES OF NONCOMPACTNESS IN THE  
STUDY OF THE SOLVABILITY OF INFINITE SYSTEMS OF INTEGRAL EQUATIONS  
DEFINED ON AN UNBOUNDED INTERVAL

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The thesis presents the study of the solvability of infinite systems of quadratic integral equations of Urysohn and Hammerstein type, which are considered on the real half-axis  $\mathbb{R}_+ = [0, +\infty)$ . Investigations are conducted in the Banach space consisting of bounded and continuous functions defined on the interval  $\mathbb{R}_+$  with values in the Banach sequence space and normalized by the standard supremum norm.

The main results of the thesis are contained in Theorem 3.5, 4.4 and 5.2. In the proofs of those theorems there are used two basic fixed point theorems (Schauder fixed point principle and Darbo fixed point theorem) and the technique associated with measures of noncompactness.

In the third chapter we will discuss the solvability of an infinite system of quadratic Urysohn integral equations in the Banach space  $BC(\mathbb{R}_+, c_0)$  consisting of bounded and continuous functions defined on the real half-axis with values in the space of real sequences converging to zero. Namely, we will consider the infinite system of the quadratic Urysohn integral equations having the form

$$x_n(t) = a_n(t) + f_n(t, x_1(t), x_2(t), \dots) \int_0^\infty u_n(t, \tau, x_1(\tau), x_2(\tau), \dots) d\tau \quad (1)$$

for  $t \in \mathbb{R}_+$  and for  $n = 1, 2, \dots$ .

In the next chapter we will investigate the solvability of the infinite system of integral equations of Urysohn type (1) in the Banach space  $BC(\mathbb{R}_+, l_\infty)$  of functions which are

continuous and bounded on the interval  $\mathbb{R}_+$  with values in the classical Banach sequence space consisting of real bounded sequences. In the proof of the presented theorem we use a suitable measure of noncompactness in the space  $BC(\mathbb{R}_+, l_\infty)$ . Due to this measure we are in a position to obtain the result on the asymptotic stability of solutions of the considered system.

The fourth chapter is devoted to present a result on the existence of asymptotically stable solutions of an infinite system of quadratic integral equations of Hammerstein type having the form

$$x_n(t) = a_n(t) + f_n(t, x_1(t), x_2(t), \dots) \int_0^\infty g_n(t, \tau) h_n(\tau, x_1(\tau), x_2(\tau), \dots) d\tau, \quad (2)$$

where  $t \in \mathbb{R}_+$  and  $n = 1, 2, \dots$ .

Our study will be in the Banach space  $BC(\mathbb{R}_+, l_\infty)$  which was described above.

In the thesis we pay attention to the fact, that infinite systems of integral equations are important in many applications. The last chapter is dedicated to formulate an open problem concerning the existence of solutions of an infinite system of nonlinear integral (or differential) equations which are obtained during the modelling of the so-called birth-and-death stochastic process.