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Abstract of the doctoral thesis

Selected properties and applications of quantile functions

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The doctoral thesis concerns issues related to quantile functions and quantile estimation, with particular emphasis on their asymptotic properties and their applications to the construction of distribution function estimators for selected random variables. Quantile functions provide an alternative characterization of probability distributions and play an important role in both theoretical and applied statistics. They have numerous applications, including risk analysis, economics, finance, reliability theory and the modeling of data with non-standard probability distributions.

The thesis focuses on two main research problems. The first concerns the generalization of the classical Bahadur representation for quantile estimators, while the second addresses the construction of approximation formulas and estimators of distribution functions for selected functions of random variables using quantile functions.

Within the first research problem, the Bahadur representation was derived for eight different definitions of the quantile estimator under four different dependence structures of random variables. Furthermore, for selected dependence structure, the uniform asymptotic normality of the quantile estimator was established. The theoretical findings were supported by simulation studies illustrating the asymptotic properties of the quantile estimators. In addition, the analysis was extended to the framework of sublinear expectation space (the space under probability uncertainty) in the case of independent random variables, where an analogue of the

classical Bahadur representation was obtained. Establishing this result required the development of new analytical tools based on the specific properties of sublinear expectations and the distinct structure of the sublinear expectation space.

The second research problem concerns the application of quantile functions to the approximation and estimation of the distribution functions of selected functions of random variables. Approximation formulas were derived for the distribution functions of the difference, product and quotient of independent random variables, which extend previous results in the literature concerning the sum of random variables. The proposed approximation formulas are based on quantile functions and the numerical trapezoidal rule. Approximation errors were established for the proposed formulas and new nonparametric estimators of the corresponding distribution functions were constructed. Moreover, the strong consistency of these estimators was proved and in selected cases their exact distributions were derived. The properties of the proposed estimators were illustrated through computer simulations by analysing approximation error, mean squared error, and bias.

The results obtained in this thesis generalize the classical theorems concerning the Bahadur representation and extend the applicability of quantile functions to the estimation of probability distributions. The proposed methods may find applications in further theoretical research as well as in practical problems arising in data analysis, reliability theory, economics, and finance.