Rzeszow University of Technology
The Faculty of Mathematics and Applied Physics

Rzeszów, 18.10.2026

Abstract of the doctoral thesis

Title: Solutions of nonlinear quadratic Hammerstein–Stieltjes integral equations converging to finite limits at infinity

Author: mgr Justyna Szczupiel
Supervisor: prof. dr hab. Józef Banaś

The doctoral thesis is devoted to the study of the existence of solutions of nonlinear quadratic Hammerstein–Stieltjes integral equations defined on the real half-axis $\mathbb{R}_+ = [0, \infty)$. The main aim of the thesis is to establish conditions guaranteeing the existence of solutions of the considered equations and to investigate their behavior at infinity. In particular, the existence of solutions converging to zero and solutions converging to finite limits, not necessarily equal to zero, is investigated.

The investigations are conducted in the Banach space $BC(\mathbb{R}_+)$, consisting of all real-valued continuous and bounded functions defined on $\mathbb{R}_+$. The main tools used in the thesis are suitably chosen measures of noncompactness and the Darbo fixed point theorem.

The initial chapters of the thesis present basic concepts and results from functional analysis, the theory of functions of bounded variation, and the Riemann–Stieltjes integral. Particular attention is paid to the properties of functions of bounded variation and the Riemann–Stieltjes integral on an unbounded interval. Selected measures of noncompactness are also discussed, especially those defined in the space $BC(\mathbb{R}_+)$, which are subsequently used in the main part of the thesis.

The main results of the thesis concern the nonlinear quadratic Hammerstein–Stieltjes integral equation of the form

$$x(t) = a(t) + f(t, x(t)) \int_0^\infty g(t, s) h(s, x(s)) d_s K(t, s), \quad t \in \mathbb{R}_+.$$

First, conditions guaranteeing the existence of at least one solution of the considered equation converging to zero at infinity are established. Next, the investigations are extended to solutions converging to finite limits at infinity, not necessarily equal to zero. In both cases, the proofs are based on the application of suitable measures of noncompactness and the Darbo fixed point theorem.

The thesis also presents examples illustrating the applicability of the obtained results to particular integral equations. The adopted assumptions allow, among other things, the consideration of Riemann–Stieltjes integrals with respect to functions of bounded variation, without requiring their monotonicity.

The thesis concludes with a summary of the obtained results and an indication of possible directions for further research on the solvability of nonlinear Hammerstein–Stieltjes integral equations.