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Maria Curie-Skłodowska University

The Faculty of Mathematics, Physics and Computer Science

Rzeszów University of Technology

The Faculty of Mathematics and Applied Physics

Abstract of the doctoral thesis

"Selected geometric properties of interpolation spaces"

Author: mgr Karol Aleksandrowicz

The supervisor: prof. dr hab. Stanisław Prus

Interpolation theory is a branch of functional analysis which has applications for example in approximation theory, differential equations and in problems concerning certain inequalities. The problem of interpolation consists in constructing spaces, which lie, in a sense, between two given compatible spaces and which have the property that each linear operator acting continuously on both these spaces is also continuous when considered as an operator defined on an interpolation space.

One of important questions concerning interpolation theory is whether properties of operators are preserved under interpolation. Another research direction concerns a problem if geometric properties of one of the spaces from pair of compatible Banach spaces transfer to an interpolation space. This dissertation is devoted to this problem.

At the beginning of the first chapter we recall definitions of geometric properties of Banach spaces, which are studied in this dissertation. In one of the interpolation methods Banach lattices are used and part of this chapter is devoted to geometric properties of lattices, in particular local versions of uniform monotonicity and uniform order smoothness.

In the second we present new results concerning general discrete interpolation method. They show that under appropriate assumptions, such geometric properties as strict convexity, local uniform convexity, near uniform convexity and property (β) transfer from one of the spaces from the compatible pair of Banach spaces to an interpolation space. In order to prove the theorem about stability of property (β), we also prove a theorem about property (β) for direct sums of Banach spaces.

The third chapter of this dissertation is devoted to Yoshikawa-Sparr interpolation method, which is a version of the real method of interpolation for more than two spaces. New results in this chapter concern stability of uniform convexity, near uniform convexity and property (β) under discrete version of this method. Next we present an

example which shows that interpolation spaces obtained by the continuous and discrete version of Yoshikawa-Sparr interpolation method can be non-isometric and therefore we need to prove theorems concerning stability of geometric properties separately for each version of this method. However, we present a method which enables us to obtain results concerning stability of geometric properties under continuous version of Yoshikawa-Sparr interpolation method using previously proved theorems concerning discrete version of the method. This new method is applied to prove theorems concerning stability of uniform convexity, near uniform convexity and property (β) under continuous version of Yoshikawa-Sparr interpolation method.

The last, fourth chapter of this dissertation is devoted to the complex method of interpolation. New results presented in this chapter concern stability of strict convexity and local uniform convexity. We also present partial results concerning stability of infinite-dimensional counterparts of uniform convexity: near uniform convexity and property (β). Later we consider the case of interpolation of Banach lattices and we prove a theorem concerning stability of uniform monotonicity.