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Abstract of the doctoral thesis

Title: *Independent and dominating sets in graphs including the set of leaves*

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The thesis investigates independent sets and dominating sets including the set of leaves, with particular emphasis on counting problems.

Independent sets including the set of leaves were previously counted in trees and unicyclic graphs. The thesis continues the study of counting independent sets including the set of leaves for a particular class of bicyclic graphs in which the set of leaves is nonempty and the two cycles have exactly one common vertex. For this class of graphs, the minimum and the maximum number of independent sets including the set of leaves were determined, and extremal graphs with respect to this number were characterized.

The thesis also considers independent $(1, 2)$-dominating sets including the set of leaves. Using a graph interpretation of the Padovan and Perrin numbers related to counting independent $(1, 2)$-dominating sets in paths and cycles, new binomial formulas for Padovan and Perrin numbers were obtained. Moreover, the Padovan polynomial of a graph was defined using the composition of two graphs.

Furthermore, the thesis studies distance k-independent sets for $k \geqslant 2$. By applying a graph interpretation of the generalized Padovan sequence, relationships between generalized Padovan numbers and the number of maximal k-independent sets including the set of leaves in paths were established. This interpretation led to a binomial formula for generalized Padovan numbers. Using the G-join operation of graphs, a generalized Padovan polynomial was defined.

Finally, the thesis investigates 2-dominating sets including the set of leaves. Since every 2-dominating set is also a 3-dominating set, proper 2-domination was introduced as a new type of 2-dominating set. Proper 2-dominating sets were defined in order to distinguish 2-dominating sets from 3-dominating sets. A complete characterization of graphs admitting a proper 2-dominating set was obtained. Furthermore, relationships between the parameters of proper 2-domination and those of 2-domination and 3-domination were established. It was also shown that the proper 2-domination number is related to the existence of independent 2-dominating sets.